A SIMPLE METHOD FOR SOLVING SIMPLE AND COMPLEX SYLLOGISMS

I. TRADITIONAL LOGIC AS A BRANCH OF EDUCATION1.

Has Traditional Logic come out of date over the last 122 years since the publication of John Venn’s Symbolic Logic?

As it might be expected, it would be difficult to stop Tradition in a discipline such as Traditional Logic. This is so because it is founded upon a perfect theory of inference: Syllogistics. In spite of constant attempts to present it from a contemporary point of view, Syllogistics remains an indispensable method for natural reasoning in a natural language, and it has important real-life applications.

The standard topics included under Traditional Logic are: Concept, Proposition, Reasoning, Proof, and so on – all that serves to provide us with practical skills for inferencing truths from other truths. There is no doubt that the main theory of inference in Traditional Logic is Syllogistics2.

In the logic textbooks syllogisms are still the object of educational interest and respect. Even though these disciplines are now often named “Critical Reasoning” or “Critical Thinking”3, they invariably include solving syllogisms. The methods which are usually used are Venn-diagrams and rules for solving syllogisms. Syllogisms are also included in the Critical Reasoning sections of the standardized tests for higher education in the USA (such as SAT, GMAT, CAT, GRE).

For these reasons using a simple and quick method for solving syllogisms would be of both educational and practical use.

---

1 This heading is borrowed from John Venn: Symbolic Logic. London, Macmillan and Co., 1881, Cambridge, p. XXV.
3 See also G. L. Tulchinski, Perspectives of Logic in Humanitarian Education, 2000, pp. 130-131 (Г. Л. Тульчинский. Перспективы логики в гуманитарном образовании) – In: Современная логика: проблемы теории, истории и применения в науке. Изд. Санкт-Петербургского Университета, 2000, с. 130-131)
II. METHODS FOR SOLVING SYLLOGISMS:

1. **Aristotle’s method** can be found in Aristotle’s main logical writings, the *Analytics*: a perfect syllogism is one whose inference causes no doubt. These are the modes of the First Figure, where the inference is simple and obvious. The other figures (and their modes) are reduced to the First Figure (and its modes), where the inference is obvious.

2. **Rules for solving syllogisms.** During the Middle Ages the detailed analyses of logicians result in well structured systematic courses in logic and often well developed modal syllogistics;

3. **Euler’s Circles,** named after Leonard Euler (1707-1783), although circles were used before him, for example by Gottfried W. Leibniz (1646-1716);

4. **Venn’s Diagrams,** named after John Venn (1834-1923);

5. **Lewis Carroll’s diagrams** (1832-1898); Lewis Carroll’s method of subscriptions;

6. **Language and methods of contemporary logic:** the language of propositional and predicate logic; axiomatic method, natural deduction and so on.

III. DEVELOPING THE TRADITIONAL METHOD FOR SOLVING SYLLOGISMS THROUGH RULES:

Only three rules for solving syllogisms are necessary and sufficient:

1. *The middle term must be distributed at least once;*

2. *The end terms are distributed in the conclusion, if they are distributed in the premises;*

3. *The number of negative conclusions must equal the number of negative premises.*

The application of these rules can be demonstrated with an example, by checking the validity of the following syllogism:

*Some Alphas are not Gammas.*

*All Betas are Gammas.*

? *Some Alphas are not Betas.*

---


2. In I. M. Bochencki, *A History of Formal Logic* (Chelsea Publishing Company, New York, 1970 (Co. 1956 in München)) there is a facsimile showing that Leibniz also used circles for solving syllogisms. In it there are the solutions to modes Camestres (A E E), Camestro (A E O), Festino (E I O), Baroco (A O O) from Second Figure; Darapti (A I I), Felapton (E A O), Disamis (I A I), Datisi (A I I), Bocardo (O A O), Ferison (E I O) from Third Figure; Camenes (A E E) from Fourth Figure (p. 260-261).


88
To the right of the syllogism its standard symbolic form is given (in the language of traditional logic: S, P, M; A, E, I, O). After the respective term we note whether it is distributed\(^9\). Then we apply the rules: where the rule is satisfied we write (+). The syllogism can even be solved without using the standard symbolic form (just by using natural language):

1. The middle term “Gammas” is distributed in the first premise – with this, rule 1 is satisfied;
2. The end terms are joined in the conclusion: “Alphas” and “Betas”. Of them, only “Betas” is distributed in the conclusion – we look at the premises and check whether it is distributed there: it is (“All Betas”). “Alphas” is not distributed in the conclusion, so we do not look for it in the premises;
   The objective of this check is to be sure that there is nothing more in the conclusion than what was in the premises. With that, the second rule is satisfied.
3. The third rule is also satisfied: there is one negative proposition in the premises and one negative conclusion.

The three rules are satisfied so we can conclude that this mode of the syllogism is valid. We write down OAO-2: Valid mode.

IV. THE TRADITION IN SYLLOGISTICS:

1. There are seven or eight rules for solving syllogisms dating back to the Middle Ages. They are correct and solve syllogisms, but they are redundant – something which has been overcome in the later development of the Tradition.
2. In his *Elementary Lessons in Logic* (1870) Jevons suggests eight rules for solving syllogisms\(^{10}\):

1) Each syllogism has three and only three terms;
2) Each syllogism consists of three and only three propositions;
3) The middle term should be distributed at least once and should not be ambiguous;
4) No term which has not been distributed in any of the premises should be distributed in the conclusion;
5) No conclusion can be derived from negative premises;
6) If one of the premises is negative then the conclusion should also be negative, and vice versa: to arrive at a negative conclusion, one of the premises should also be negative;
7) No conclusion can be drawn from two particular premises;
8) If one of the premises is particular then the conclusion should also be particular.

---

\(^{8}\) This is the standard symbolic form of propositions in Traditional Logic, used to this day.


Of these eight rules, the first two are not rules in the strict sense of the word, but simply define the nature of syllogisms. As a result, there remain only six rules for solving syllogisms. What is more, the last two rules (7 and 8) are not independent, and Jevons illustrates how they are derived from the other rules\footnote{W. St. Jevons (Elementary lessons in logic. London 1870) (translation by Ekaterina Karavelova), Plovdiv, 1884, c. 134-135.}. So all in all, four rules remain in the end.

3. In William Spalding’s *Introduction to Logical Science* (Edinburgh, 1857), there are only 6 rules:

1) A Syllogism has Three Terms;
2) The Middle term must be Distributed in one of the premises;
3) Neither the Minor term nor the Major must be Distributed in the Conclusion, if it was Undistributed in its Premise;
4) If both Premises are Affirmative, the Conclusion must be Affirmative;
5) If either of the Premises is Negative, the Conclusion must be Negative;
6) From Premises, both of which are Negative, no Conclusion must be inferred. (pp. 206-209).

The first rule refers to the nature of the syllogism and need not be considered as a rule. Rules 4, 5 and 6 can be reduced to a single rule: “The number of negative conclusions must equal the number of negative premises”. This is the third rule of the three rules for solving simple categorical syllogisms. That is all that is needed for solving a syllogism: rules 2, 3 and the common rule about negative propositions in the syllogism.


1) A syllogism must contain no more and no less than three terms;
2) The middle term must be distributed at least in one of the premises;
3) The end terms are distributed in the conclusion, if they are distributed in the premises;
4) No conclusion can be derived from negative premises;
5) If one of the premises is negative then the conclusion (if it is possible) is also negative.

The first rule refers to the nature of the syllogism and need not be considered as a rule.

The second rule is accurate and flawless: the middle term must be distributed at least once.
So is the third rule – in the conclusion, where S and P are connected, their extensions must not exceed their extensions in the premises.

The fourth and fifth rules can be joined in a single rule: the number of negative conclusions and premises is equal (0,0; 1,1).

Thus we arrive at the three simple rules for solving syllogisms. If only we had not ignored Tradition and Evolution in the development of Traditional Logic.

V. USING THE METHOD OF THE THREE RULES, WE CAN NOT ONLY CHECK THE VALIDITY OF A GIVEN SYLLOGISM BUT ALSO FIND A CONCLUSION:

Let us look at a “simple” syllogism which is not so simple: what conclusion can be derived from the two premises below?

Some philosophers are logicians.  
No mathematician is a philosopher.  
\therefore Some logicians are not mathematicians.

We apply the rules in the following sequence:

1) Md – satisfied;
2) The number of negative conclusions equals the number of negative premises;
3) The end terms S and P are distributed in the conclusion, if they are distributed in the premises; that is, in the conclusion we can have Sd, because it is present in the premises. There can be no E-conclusion, because we do not have two distributed terms (Sd and Pd) in the premises. The only possible solution is an O-conclusion. In order to have Sd in the conclusion, it has to be a predicate of an O-proposition. So the conclusion is POS.

This is how this syllogism can be solved without standard symbolic script:

Some philosophers are logicians.  
No mathematician is a philosopher.  
\therefore Some logicians are not mathematicians.

“Philosopher” is the Middle Term connecting both premises. According to the First Rule, it has to be distributed at least once. This is satisfied in the second premise, which is an E-proposition: in it the two terms are distributed, that is, the term “philosopher” is distributed (the term is in its full extension).

According to the rules, when one of the premises is negative, the conclusion is negative. The other two terms – “mathematicians” and “logicians” are connected in the conclusion in a negative proposition. However, “logicians” is not distributed, because we have an I-proposition. That means that we cannot have an E-proposition
in the conclusion (in an E-proposition both terms are distributed). So the only possibility is an O-proposition.

“Mathematicians” is taken in its full extension (distributed term) in the premises, so it can also be distributed in the conclusion. The conclusion will be an O-proposition with “mathematicians” in the right-hand part of the O-proposition (where the distributed terms of the O-proposition are placed). That is, ∴ “Some logicians are not mathematicians”.

VI. SORITESES. A GENERAL THEORY FOR SOLVING SYLLOGISMS.

Syllogisms with more than two premises (a chain of premises) are referred to as poly-syllogisms, complex syllogisms, or soriteses. J. T. Culbertson claims that Aristotle himself looked at such complex syllogisms.13

Complex syllogisms can be solved by reducing them to simple syllogisms: the premises are solved in pairs. There could be a common method for solving them, however. J. T. Culbertson gives the following rules for solving soriteses:

**Rule 1.** Only the last premise can be negative and only the first premise can be a particular proposition.

**Rule 2.** The premise is negative when the conclusion is negative.

**Rule 3.** If one of the premises is a particular proposition, the conclusion is also a particular proposition.14

Let us analyze these rules. Why can only the last premise be negative? The order of premises is irrelevant in the simple syllogism. But let us see:

No monkey is a human.
All humans are mortal.
All mortals are living beings.

We can solve this complex syllogism if we know how to solve simple syllogisms. We have the three rules for the simple categorical syllogism:

**Rule 1.** M, the middle term, must be distributed at least once;

**Rule 2.** The end terms S and P are distributed in the conclusion, if they are distributed in the premises;

**Rule 3.** The number of negative conclusions must equal the number of negative premises.

These are also the rules given by Culbertson15 (even if not in the same order) — they are an exact and minimal method for solving the simple syllogism:

---

No monkey is a human.  
All humans are mortal.  
\[\therefore\] Some mortals are not monkeys.

We can add the third premise to this conclusion and will arrive at a simple syllogism again:

Some mortals are not monkeys.  
All mortals are living beings.  
\[\therefore\] Some living beings are not monkeys.

The simplest structure of the complex syllogism is a chain carrying the information from the first to the last term: P – M1 – M2 – M3 – … – S.

It turns out that the first premise can be negative and any other premise can be negative, but there can be only one negative premise in each sorites.

Rule 1 also says that only the first premise can be a particular proposition. This is rather strange and can be checked:

All mammals are warm blooded.  
All bats are mammals.  
Some bats eat ripe figs\(^{16}\).  
\[\therefore\]…………………………

Let us solve this sorites step-by-step. Step one:

All mammals are warm blooded.  
All bats are mammals.  
\[\therefore\] All bats are warm blooded.

And step two:

All bats are warm blooded.  
Some bats eat ripe figs.  
\[\therefore\] Some warm blooded eat ripe figs.

From the example above it is obvious that the second part of the first rule is also wrong: a complex syllogism can have a particular proposition as a last premise.

The second rule is correct, but in the reverse: “The conclusion is negative when the premise is negative”. Or, even more correctly, “The number of negative conclusions equals the number of negative premises” – a rule which we have inherited from the simple categorical syllogism.

\(^{16}\) There are fruit-eating bats on the isle of Java who eat ripe figs.
The third rule is redundant. A poly-syllogism cannot have two negative premises, because nothing follows from them (according to one of the rules of the simple syllogism): the chain of syllogism is broken.

In the same way, we cannot have two particular premises in a poly-syllogism, because nothing follows from them (as with the simple syllogism) and the chain is broken. So we can have only one particular premise. Culbertson’s third rule says: “If one of the premises is a particular proposition, the conclusion is also a particular proposition”. This is true, but it is redundant for the same reason that it is redundant for the simple syllogism:

Here “d” means that a given term is distributed: the Subject of the A-proposition is distributed; both the Subject and Predicate of the E-proposition are distributed; and only the Predicate of the O-proposition is distributed.

If we look at the four possible cases of the existence of a particular premise, we shall discover the following:

1) In the first case we have only one distributed term above the line and we take it as a Middle Term, which means that in the conclusion we can have neither A-proposition nor E-proposition. If we apply the rule for the equal number of negative propositions above and below the line, it will turn out that we cannot have O-proposition either. For these reasons we take an I-proposition for a conclusion;

2) In the second case because of the rule of the equal number of negative propositions above and below the line there can be only negative propositions in the conclusion – E-proposition or O-proposition. But we cannot have an E-proposition because we need to have three distributed terms in the premises (two for the E-proposition and one for the Middle Term), which is not the case. The only possibility left is an O-proposition in the conclusion; its Predicate is the distributed term;

3) The reasoning in the third case is analogous as that in the second case;

4) In the fourth case we simply cannot have a conclusion. There is always one conclusion, and according to the rule, the number of negative premises and conclusions is equal (1=1).

But we necessarily need another rule, as with the simple categorical syllogism: the end terms in the conclusion cannot have larger extension than in the premises. The poly-syllogism is a multiplied simple syllogism. Thus the rules for solving complex syllogisms are three again:
Rule 1. Every middle term in the complex syllogisms, $M_1, \ldots, M_n$, must be distributed at least once.
Rule 2. The end terms $S$ and $P$ are distributed in the conclusion, if they are distributed in the premises;
Rule 3. The number of negative conclusions must equal the number of negative premises.

The second and third rules are absolutely the same as in the simple syllogism: if we know the logical explanation of these rules it is easy to see why they do not need to be changed. Only the first rule is somewhat applicable to the needs of the complex syllogism where we have more than one Middle Term. And since the middle terms establish the link between the premises, each needs to be distributed at least once.

Let us now solve the same syllogism directly:

\[
\begin{align*}
\text{All mammals are warm-blooded.} & \quad M_1d \quad A \quad P \\
\text{All bats are mammals.} & \quad M_2d \quad A \quad M_1 \\
\text{Some bats eat ripe figs.} & \quad M_2d \quad I \quad S \\
\therefore \text{Some warm-blooded animals eat ripe figs.} & \quad \therefore \quad P \quad I \quad S
\end{align*}
\]

Let us take one of Lewis Carroll’s soriteses and try to apply the rules:

1. My saucepans are the only things I have that are made of tin;
2. I find all your presents very useful;
3. None of my saucepans are of the slightest use.\(^\text{17}\)

First we find the Middle Terms:
M\(_1\) – my saucepans (1 \& 3 proposition);
M\(_2\) – useful (2 \& 3 proposition).

\[
\begin{align*}
1. \quad M_1d & \quad \text{My saucepans are the only things made of tin} \quad P \\
\quad \text{(M\(_1\)AP)} & \\
2. \quad Sd & \quad \text{all your presents very useful} \quad M_2 \\
\quad \text{(S A M\(_2\))} & \\
3. \quad M_1d & \quad \text{my saucepans are not useful} \quad M_2d \\
\quad \text{(M\(_1\)EM\(_2\))} &
\end{align*}
\]

This is the “translation” of the sorites into the language of Logic (Syllogistics): with clearly distinct Subjects, Predicates, Middle Terms. Next we apply the rules for solving syllogisms:

1. \( \text{MI or M2 must be distributed at least once} \) – satisfied;
2. \( \text{The number of negative conclusions must equal the number of negative premises.} \)
   That means that conclusion is E or O-proposition.
3. \( \text{The end terms in the conclusion must not more extended than in the premises;} \)
   that is, S and P are distributed in the conclusion if they have been distributed in the premises. In the premises they are in their full extension, which means that they can be in their full extension in the conclusion too, so we can have an E-proposition.

   We shall designate the two terms which are present only once in the premises of the sorites as S and P. Proposition (1) is an A-proposition, (“My saucepans are the only things I have that are made of tin”), but the expression “only” shows that the Predicate of that proposition is also distributed.

   The Subject of the second proposition (“I find all your presents very useful”) is “your presents” and it is distributed: “all your presents”. As both end terms are distributed (taken in their full extension), they can also be distributed in the conclusion, that is, we can have an E-proposition, which is reversible: S E P becomes P E S:

   \[ \text{None of my things made of tin is your present.} \]
   \[ \text{None of your presents is among my things made of tin.} \]
   \[ \text{(In a free linguistic form: Your presents to me are not made of tin.)} \]

This is the short way of arriving at a conclusion from the premises of the complex syllogism. In the example given, the Predicate can also be distributed in the SAP-proposition. For instance, “Only people are reasonable beings”.

However, it would be wrong to believe that syllogistics consists only of the three simple rules for solving syllogisms. Before that, we should be acquainted with the traditional logical subject matter: concepts, “addition of concepts”, propositions, transformation of propositions (conversion, obversion, contraposition); negation, types of negation, the bivalent principle in logic, and so on. And even before that, we should be able to “translate” from ordinary language into the language of logic, in this case – the language of traditional syllogistics. Let us look at some other soriteses by Lewis Carroll:

Sorites № 5.
1. No ducks waltz;
2. No officers ever decline to waltz;
3. All my poultry are ducks.\(^\text{18}\)

Here is a direct solution to this sorites:

1. No ducks waltz.          M1d E M2d
2. Every officers waltz.  Pd A M2
3. All my poultry are ducks.  Sd A M1
   ∴ None of my poultry are officers.  Sd E Pd
   ∴ The officers are not my poultry.  Pd E Sd

Or, put in a freer linguistic form:

∴ My poultry are not officers.
∴ The officers are not my poultry.

Sorites № 13.
1. All humming birds are richly coloured;
2. No large birds live on honey;
3. Birds that do not live on honey are dull in colour.19

The expression “do not live on honey” is encountered in two of the premises and is a Middle Term, M1 (more exactly “birds that do not live on honey”). As M2 we take the expressions “richly coloured” and “dull in colour”. We take one of the two as a basic term, and the other is expressed through the negation of the first – for example “All humming birds are richly coloured” and “Birds that …, are not richly coloured”. And then the scheme of the syllogism is:

1. All humming birds are richly coloured.  Pd A M2
2. No large birds live on honey.  Sd A M1
3. Birds that do not live on honey are dull in colour.  M1d E M2d
   ∴ No humming birds are large birds.  Pd E Sd
   ∴ No large birds are humming birds.  Sd E Pd

Visually presented, the scheme of the syllogism is:

1. Pd    humming birds +    richly coloured    M2
2. Sd    large birds +    do not live on honey    M1
3. M1p   birds that don’t live on honey –    richly coloured    M2p

∴ P E S (No humming-bird is a big bird)
∴ or also S E P (No big bird is a humming bird).

The rules for solving the sorites are fulfilled:
1. $M_1$ and $M_2$ are distributed at least once – both are distributed in third premise.
2. $S$ and $P$ are distributed in the premises – so we can have them distributed in the conclusion.
3. The number of negative premises and conclusions is equal ($1=1$).

Sorites № 15.
1. All ducks in this village that are branded “B”, belong to Mrs. Bond;
2. Ducks in this village never wear lace collars, unless they are branded “B”;
3. Mrs. Bond has no gray ducks in this village.\( ^{20} \)

The scheme of the syllogism is:

\[
\begin{array}{c}
M_1d & A & M_2 \\
/ \\
M_1d & A & P \\
/ \\
M_2d & E & Sd \\
/ \\
Pd & E & Sd \\
Sd & E & Pd \\
\end{array}
\]

$\therefore$ Ducks that wear lace collars are not gray ducks.

Sorites № 16.
1. All the old articles in this cupboard are cracked;
2. No jug in this cupboard is new;
3. Nothing in this cupboard, that is cracked, will hold water.\( ^{21} \)

1.  

\[
\begin{array}{c}
\text{M1p (All)} \\
\text{The old articles in this cupboard} \\
\text{are} \\
\text{cracked (things)} = M_2
\end{array}
\]

2.  

\[
\begin{array}{c}
\text{Pp (All)} \\
\text{The jugs in this cupboard} \\
\text{are} \\
\text{Old articles} = \text{are} = M_1 \\
\text{not new articles}
\end{array}
\]

3.  

\[
\begin{array}{c}
\text{M2p (All)} \\
\text{The cracked articles in this cupboard} \\
\text{will not} \\
\text{hold water} = Sp
\end{array}
\]

$\therefore$ No jug in this cupboard will hold water.

\( ^{20} \) L. Carroll. Ibid, p. 1122.

\( ^{21} \) L. Carroll. Ibid, p. 1122.
An old trick is to exchange the order of premises in the sorites:

1. All the old articles in this cupboard are cracked;
2. No jug in this cupboard is new;
3. Nothing in this cupboard, that is cracked, will hold water.

If we want to have strict sequence of the propositions (visible transitivity), we would order them in the following way:

1. No jug in this cupboard is new;
2. All the old articles in this cupboard are cracked;
3. Nothing in this cupboard, that is cracked, will hold water.

The Middle Terms in this sorites are: “cracked” (cracked articles) and “old articles”. But then the second proposition, even though it appears to be negative at first sight, is treated as positive: “All jugs in this cupboard are old (old articles)”. Here there also seem to be two negative propositions in the premises, while indeed there is only one negative proposition. The other two terms are end terms and they are connected according to the rules of the syllogism in the conclusion:

\[ \therefore \text{No jug in this cupboard will hold water.} \]

**VII. THE ADVANTAGES OF THIS METHOD OF SOLVING SYLLOGISMS ARE:**

1) These three rules are applicable to all 256 possible schemes of inference (possible syllogistic schemes) of the simple categorical syllogism. They are necessary and sufficient for rejecting the invalid modes of the syllogism, as well as for confirming their validity. In other words, no separate common and particular rules for every figure are necessary;
2) The three rules are used as a method for proving the validity or invalidity of the syllogism (see above), but also serve for establishing the conclusion, or in other words, for achieving “new” knowledge;
3) These rules can also be applied to complex syllogisms (a chain of syllogisms, known as soriteses); only one additional condition is necessary (regarding the first rule: each middle term needs to be distributed at least once);
4) These rules make determining the figure of the syllogism redundant. We do not need to know the figure and the mode in order to solve the syllogism. The figure and the mode are only necessary in writing down the name of the syllogism we have already solved, for example AAA–1, which is mode BARBARA, First figure;
5) We do not need to order the premises, for example the major premise first and the minor one second. It is of no consequence in which premise the Subject and the

---

Predicate are. In the conclusion they take their place S and P according to their extension in the premises: they cannot be in a greater extension in the conclusion;
6) The language is simple and very close to the natural language: Subject and Predicate – which is similar to the grammatical structure of natural languages;
7) The simplicity of the language and the simplicity of the method (3 rules which have a logical explanation so they do not need to be learned by heart) make it possible that the syllogisms are solved even without symbolic script.